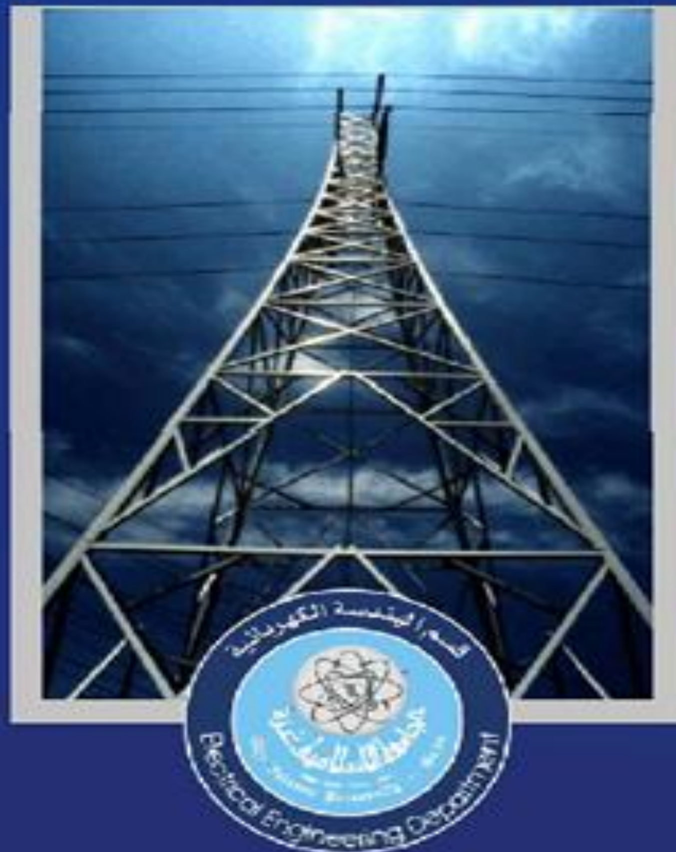
THE ISLAMIC UNIVERSITY OF GAZA



ELECTRICAL DEPARTMENT

Engineering Faculty

بسم الله الرحمن الرحيم

ELECTROMAGNETIC DISCUSSION

Chapter 1
Vector Algebra
Part One

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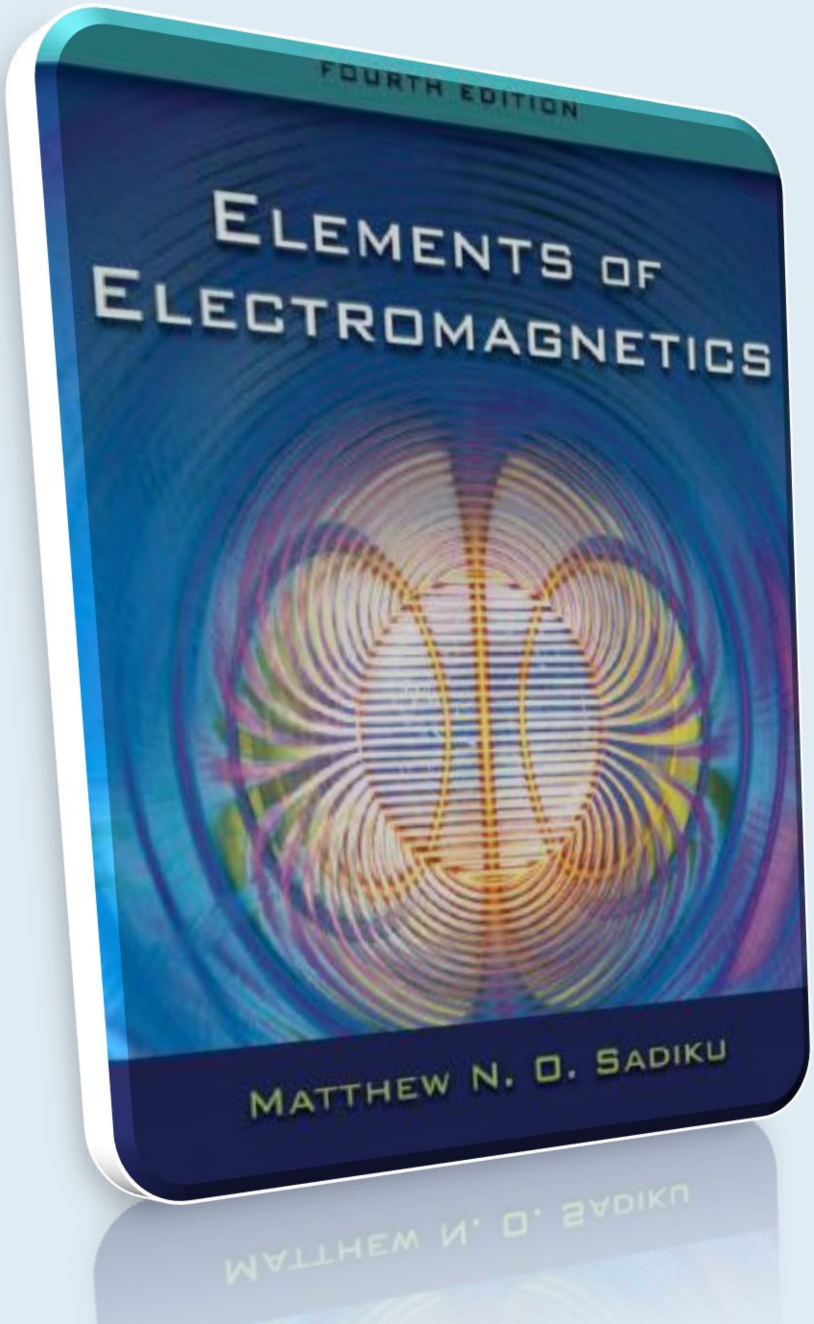


Vector Analysis

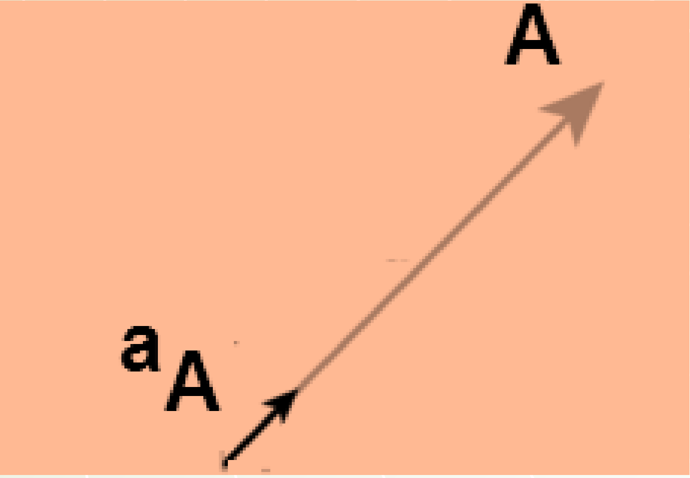
Electrostatic fields

Magnetostatic fields

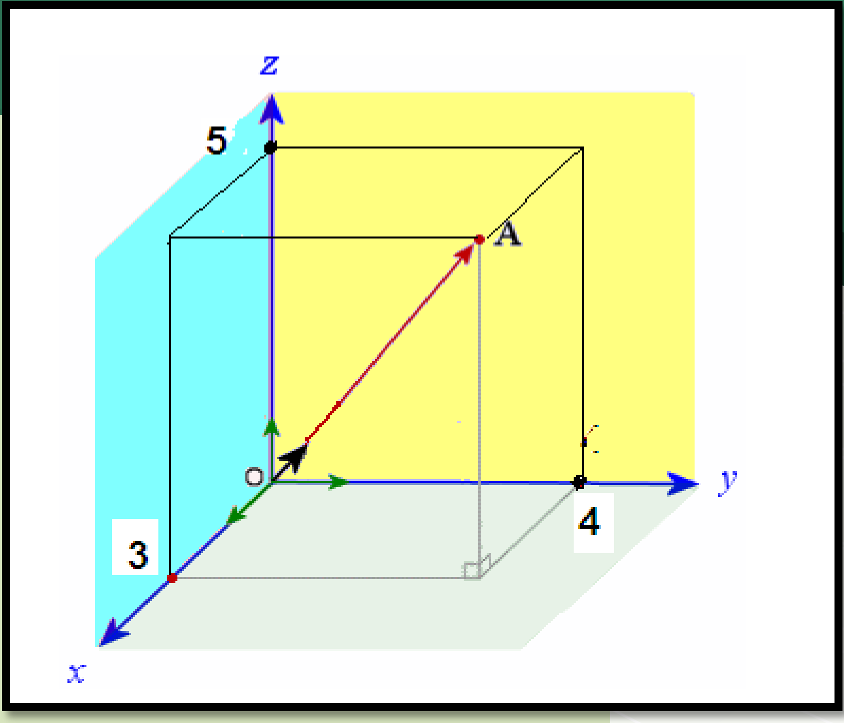
Electromagnetic fields(wave)

Unit Vector of a vector A

is a vector whose magnitude is unity and its direction is along A



Example



$$\overline{A} = 3ax + 4ay + 5az$$

$$aA = \frac{A}{|A|}$$

$$|A| = \sqrt{\left(Ax^2 + Ay^2 + Az^2\right)}$$

$$= \sqrt{\left(3^2 + 4^2 + 5^2\right)} = \sqrt{50} = 7.071$$

$$aA = \frac{3ax + 4ay + 5az}{7.071}$$

$$aA = 0.42 \; ax + 0.5657 \; ay + 0.7071 \; az$$

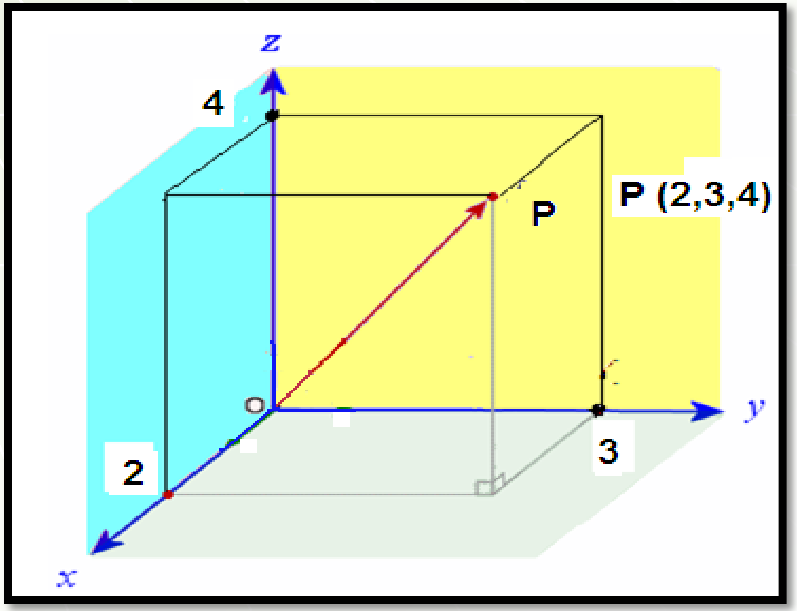
$$|aA| = 1 \; ,, magnitude = 1 ,, direction \text{ along A}$$

Position Vector of point P

is a vector directed from origin to P

e.g.

OP=P-O= 2 ax + 3 ay + 4 az



Distance Vector

directed from one point to another point .

e.g.

PQ=Q-P=

$$=(0-1)a_x+(3-2)a_y+(1-5)a_z$$

$$=-a_x + a_y - 4\ a_z$$

PROBLEM 1.1

P1(2,4,4) , P2(-3,2,2)
Find the unit vector along P1P2

$$P1P2 = P2 - P1 = (-3-2)ax + (2-4)ay + (2-4)az$$

$$= -5ax - 2ay - 2az$$

$$a_{P1P2} = \frac{P1P2}{|P1P2|} = \frac{-5ax - 2ay - 2az}{\sqrt{25+4+4}} = -0.87ax - 0.348ay - 0.348az$$

P.E 1.1

A=ax+3az
B=5ax+2ay-6az
Find:

(a) |A+B|

$$|A+B| = |6ax + 2ay - 3az|$$
$$= \sqrt{36+4+9} = 7$$

(c) The component of A along ay

zero

(d) Unit vector parallel to 3A+B

$$3A + B = 8ax + 2ay + 3az$$

$$|3A + B| = \sqrt{64 + 4 + 9} = \sqrt{77}$$

$$a_{3A+B} = \frac{8ax + 2ay + 3az}{\sqrt{77}} = 0.91168ax + 0.22799ay + 0.3419az$$

P.E 1.2

Points P(1,-3,5) , Q(2,4,6) , R(0,3,8)
Find:

(a) Position vectors for P and R

$$OP = P - O = ax - 3ay + 5az$$

$$OR = R - O = 3ay + 8az$$

(b) Distance vector QR

$$QR = R - Q = (0-2)ax + (3-4)ay + (8-6)az$$

$$= -2ax - ay + 2az$$

Points P(1,-3,5) , Q(2,4,6) , R(0,3,8)
Find:

(c) Distance between Q and R

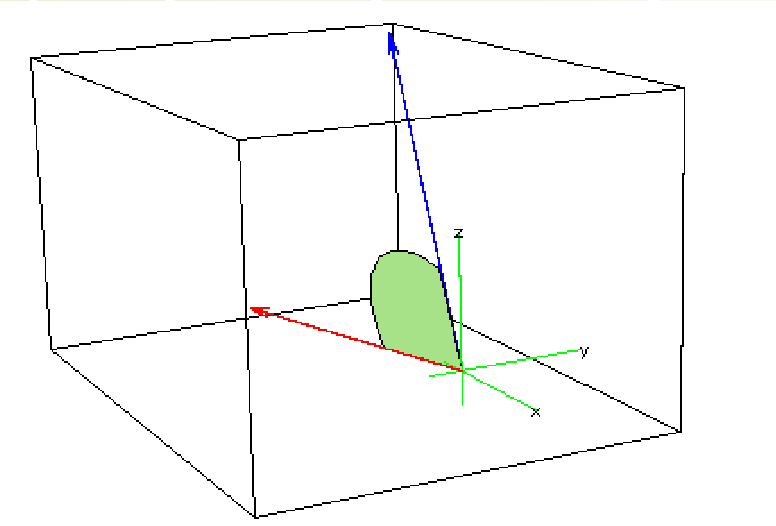
$$QR = -2ax - ay + 2az$$

$$|QR| = \sqrt{4+1+4} = 3$$

Vector Multiplication

(1) Dot Product (Scalar)

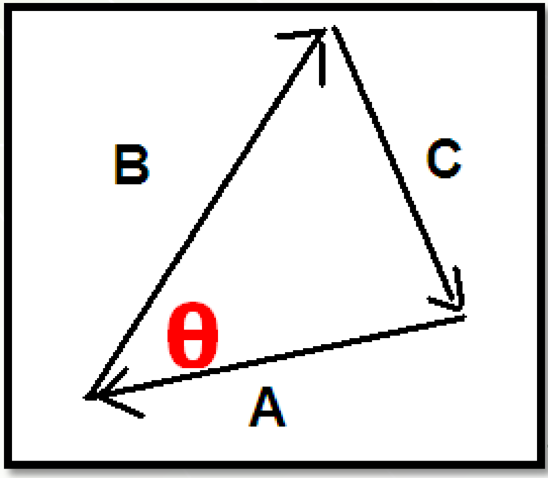
$$A = A_x.a_x + A_y.a_y + A_z.a_z$$
$$B = B_x.a_x + B_y.a_y + B_z.a_z$$



$$A.B = |A|\ |B| \cos\theta$$
$$A.B = A_xB_x + A_yB_y + A_zB_z$$

Note: →

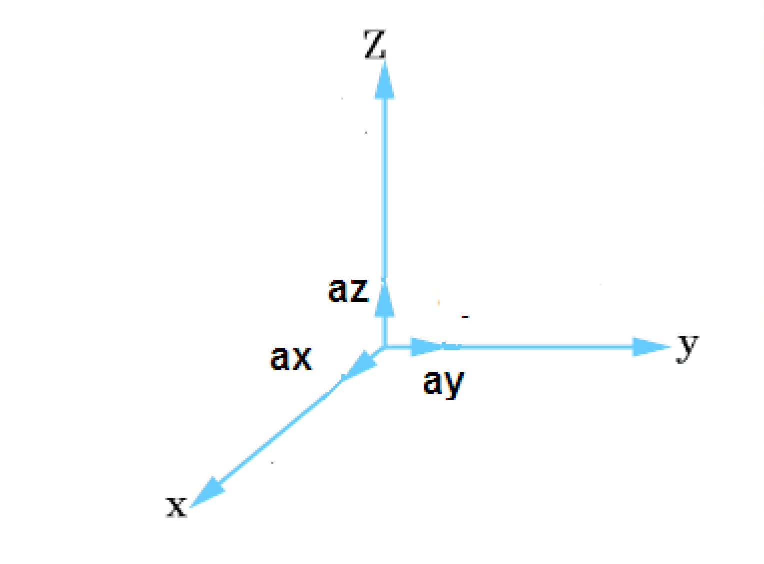
$$A.B = -|A|\ |B| \cos\theta$$



Notes:

(1) If $A.B=0 \rightarrow \theta_{AB} = 90 \rightarrow$ Orthogonal

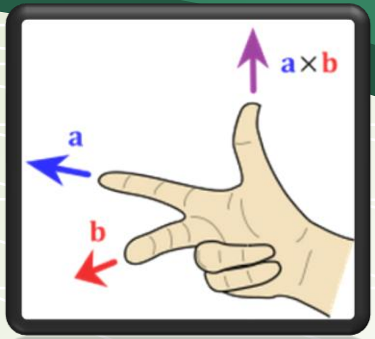
$ax.ay=0$
$ax.az=0$
$ay.az=0$



(2) $ax.ax=|ax|^2=1$
$ay.ay=|ay|^2=1$
$az.az=|az|^2=1$

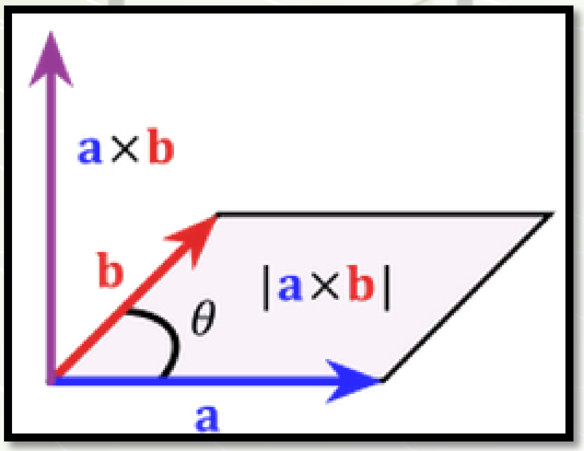
(3) $A.A=|A|\ |A| \cos 0 = |A|^2$

(2) Cross Product (Vector)



$$A = A_x.a_x + A_y.a_y + A_z.a_z$$
$$B = B_x.a_x + B_y.a_y + B_z.a_z$$

$$A \times B = |A|\ |B|\ \sin\theta\ .a_n$$



$$A \times B = \begin{vmatrix} a_x & a_y & a_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

- Cross product is a vector

direction: **Orthogonal to A and B plane**

magnitude: **area of parallelogram**

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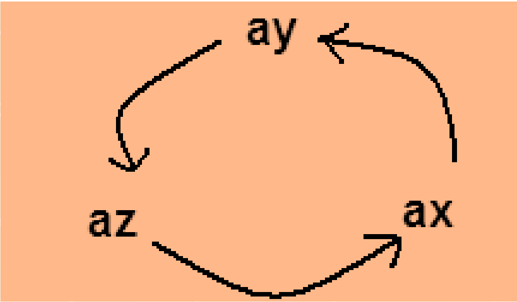
Notes:

(1) AxB=-(BxA)

(2) a_x x a_y = a_z

a_y x a_z = a_x

a_z x a_x = a_y



a_x x a_z = $-a_y$

(1) AxA=|A| |A| sin 0 = 0

e.g.

A=2ax-ay-2az

B=4ax+3ay+2az

$$AxB=\begin{vmatrix} ax & ay & az \\ 2 & -1 & -2 \\ 4 & 3 & 2 \end{vmatrix}$$

$$=[(-1*2)-(3*-2)]\,\mathrm{ax}+[(-2*4)-(2*2)]\mathrm{ay}+[(2*3)-(-1*4)]az$$

$$=4\ \mathrm{ax}-12\ \mathrm{ay}+10\ \mathrm{az}$$

(AxB).A=|AxB||A|cos 90= 0 (normal)

(AxB).B=0 (normal)

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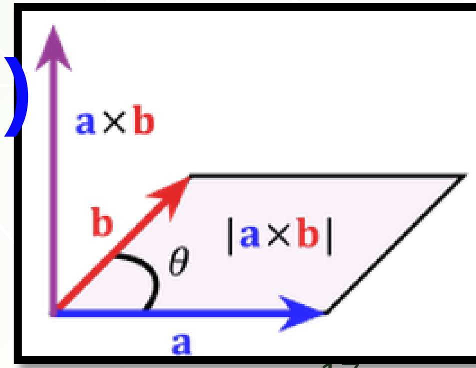


P.E 1.4

A=ax+3az
B=5ax+2ay-6az
Find the angle between vector A and B

A.B=5+0-18=-13

$$|A| = \sqrt{1+9} = \sqrt{10}$$

$$|B| = \sqrt{25+4+36} = \sqrt{65}$$

$$A.B = |A||B|\cos\theta$$

$$\cos\theta = \frac{-13}{\sqrt{10}\sqrt{65}} = -0.5099$$

$$\theta = \cos^{-1}(-0.5099) = 120.65^{o}$$

PROBLEM 1.3

A=2ax+ay-3az
B=ay-az
C=3ax+5ay+7az
Find:

(d) A.C - |B|^2

$$A.C = (2*3)+(1*5)+(-3*7) = 6+5-21 = -10$$

$$|B| = \sqrt{1+1} = \sqrt{2}$$

$$|B|^2 = 2$$

$$A.C - |B|^2 = -10-2 = -12$$

A=2ax+ay-3az
B=ay-az
C=3ax+5ay+7az

$$(d)\ \frac{1}{2}B \times \left(\frac{1}{3}A+\frac{1}{4}C\right)$$

$$\frac{1}{2}B=\frac{1}{2}ay-\frac{1}{2}az$$

$$\frac{1}{3}A=\frac{2}{3}ax+\frac{1}{3}ay-az$$

$$\frac{1}{4}C=\frac{3}{4}ax+\frac{5}{4}ay+\frac{7}{4}az$$

$$\left(\frac{1}{3}A+\frac{1}{4}C\right)=\frac{17}{12}ax+\frac{19}{12}ay+\frac{3}{4}az$$

$$\left(\frac{1}{2}ay-\frac{1}{2}az\right) \times \left(\frac{17}{12}ax+\frac{19}{12}ay+\frac{3}{4}az\right)$$

$$\left(\frac{1}{2}\,ay - \frac{1}{2}\,az\right) \times \left(\frac{17}{12}\,ax + \frac{19}{12}\,ay + \frac{3}{4}\,az\right)$$

$$\begin{vmatrix} ax & ay & az \\ 0 & \frac{1}{2} & -\frac{1}{2} \\ \frac{17}{12} & \frac{19}{12} & \frac{3}{4} \end{vmatrix} = 1.1667\,ax - 0.708\,ay - 0.708\,az$$

PROBLEM 1.5

$A=5a_x+3a_y+2a_z$, $B=-a_x+4a_y+6a_z$, $C=8a_x+2a_y$
Find α and β such that :
$\alpha A+\beta B+C$ is parallel to y-axis

$\alpha A+\beta B+C=[5\alpha a_x+3\alpha a_y+2\alpha a_z]+[-\beta a_x+4\beta a_y+6\beta a_z]+[8a_x+2a_y]$

$=(5\alpha-\beta+8)a_x+(3\alpha+4\beta+2)a_y+(2\alpha+6\beta)a_z$

➔ $5\alpha-\beta+8=0$ …..(1)
➔ $2\alpha+6\beta=0$ ……(2)
solving (1) and (2)
➔ $\alpha=-1.5$, $\beta=-0.5$

PROBLEM 1.6

A= αax+ 3ay- 2az , B=4ax+ β ay+ 8 az

(a) Find α and β if A an B are parallel?

AxB=|A||B|sin 0 =0

$$\text{AxB=}\begin{vmatrix} ax & ay & az \\ \alpha & 3 & -2 \\ 4 & \beta & 8 \end{vmatrix}$$

$= [(3*8)-(-2*\ \beta)]\ \text{ax} + [(-2*4)-(\alpha*8)]\text{ay} + [(\alpha*\beta)—(3*4)]az$

$=[24+2\beta]\ \text{ax} + [-8-8\alpha]\ \text{ay} + [\alpha\beta - 12]\ \text{az} = 0\ \text{ax} + 0\ \text{ay} + 0\ \text{az}$

➔ $24+2\beta = 0$ ➔ $\beta = -12$

➔ $-8-8\alpha=0$ ➔ $\alpha = -1$

PROBLEM 1.6

$A = \alpha a_x + 3a_y - 2a_z$, $B = 4a_x + \beta a_y + 8a_z$

(b) Relationship between α and β if A an B are perpendicular?

$A.B = |A||B| \cos 90 = 0$

$4\alpha + 3\beta - 16 = 0$

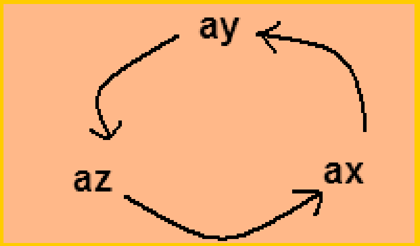
$\alpha = 0.25 - 0.75\beta$

PROBLEM 1.7

(b) Show that

$$\mathrm{ax} = \frac{\mathrm{ay \ x \ az}}{\mathrm{ax \ . \ ay \ x \ az}}$$

$$\rightarrow \frac{\mathrm{ay \ x \ az}}{\mathrm{ax \ . \ ay \ x \ az}} = \frac{ax}{ax \,. ax} =$$



$$\frac{ax}{|ax||ax|\cos 0} = \frac{ax}{|ax|^2} = \frac{ax}{1^2} = ax$$

(3) Scalar Triple Product

A.(BxC)=B.(CxA)= C.(AxB)
= volume of Parallelepiped

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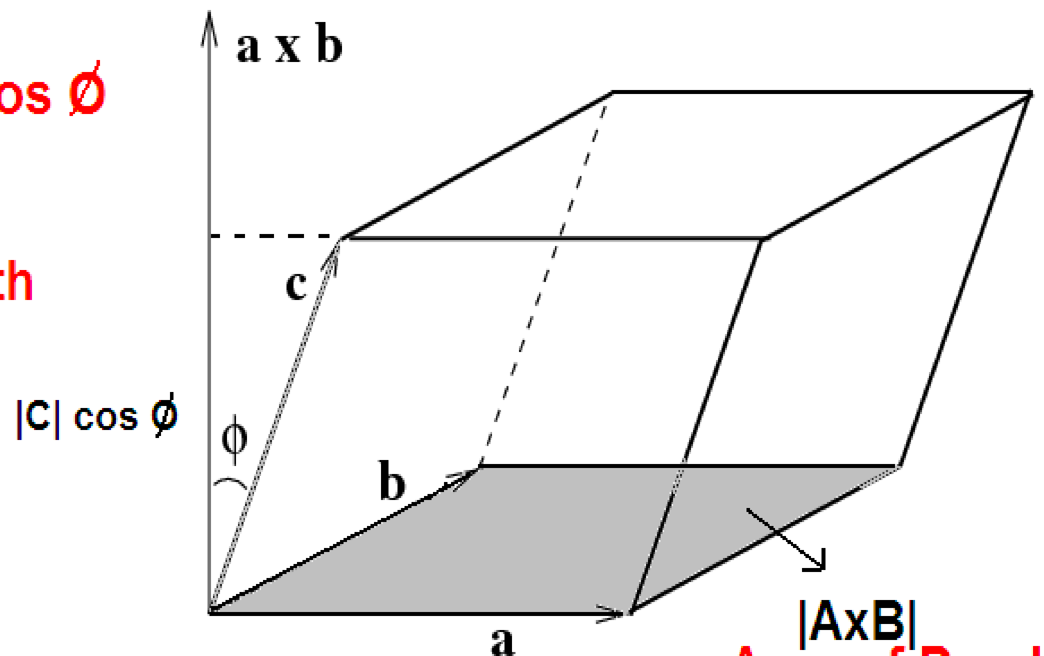
C.(AxB)=|C| |AxB| cos Ø

=|AxB| |C| cos Ø

= area of base * length



Area of Parallelogram

$$A.(B \times C) = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

PROBLEM 1.14

Show that
A.(BxC)=(AxB).C

$$A\,.\,(BxC) = \begin{vmatrix} Ax & Ay & Az \\ Bx & By & Bz \\ Cx & Cy & Cz \end{vmatrix}$$

$$(AxB).C = \begin{vmatrix} Ax & Ay & Az \\ Bx & By & Bz \\ Cx & Cy & Cz \end{vmatrix}$$

Thank You !

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